Editor's comment. As is often the case with a trigonometry problem, there were several different approaches exhibiting a variety of efficiency and recourse to other results. Kouba produced a solution similar to the second one above and Woo analyzed the graph of $y = 2\tan x - \sec x$ to show that it lay above the line $y = -\pi/3 + 2x$. Some solvers used the representation of the tangents and cosines of the half angles of a triangle in terms of the sides, semi-perimeter, inradius and area. Brown noted that the given inequality is equivalent to $s^2 \geq 12Rr + 3r^2$, while Lau reduced it to $3s^2 \leq (4R+r)^2$. Dincă proved this generalization : Let $A_1A_2\ldots A_n$ be a convex $n-$gon. Then

$$\sum_{k=1}^{n} \tan\frac{A_k}{2} \geq \cos\frac{\pi}{n}\sum_{k=1}^{n}\sec\frac{A_k}{2}.$$

3777. [2012 : 335, 336] *Proposed by G. Apostolopoulos.*

Let x, y, and z be positive real numbers such that $xyz = 1$ and $\frac{1}{x^4} + \frac{1}{y^4} + \frac{1}{z^4} = 3$. Determine all possible values of $x^4 + y^4 + z^4$.

Solved by A. Alt ; Š.Arslanagic ; D. Bailey, E. Campbell and C. Diminnie ; M. Bataille ; C. Curtis ; R. Hess ; O. Kouba ; D. Koukakis ; S. Malikić (2 solutions) ; P. Perfetti ; A. Plaza ; C. M. Quang ; D. Smith ; D. R. Stone and J. Hawkins ; I. Uchiha ; D. Văcaru ; T. Zvonaru ; and the proposer. There was also an incorrect solution. We give a solution that is a composite of virtually all solutions received.

By the AM-GM Inequality, we have

$$3 = \frac{1}{x^4} + \frac{1}{y^4} + \frac{1}{z^4} \geq 3\sqrt[3]{\frac{1}{x^4}\cdot\frac{1}{y^4}\cdot\frac{1}{z^4}} = 3.$$

Thus, we must have the equality above, which implies that $\frac{1}{x^4} = \frac{1}{y^4} = \frac{1}{z^4}$ or $x = y = z$. Since we know that $xyz = 1$, it follows that $x = y = z = 1$ and so $x^4 + y^4 + z^4 = 3$.

3778. [2012 : 335, 337] *Proposed by M. Bataille.*

Let $\Delta A_1A_2A_3$ be a triangle with circumcentre O, incircle γ, incentre I, and inradius r. For $i = 1, 2, 3$, let A_i' on side A_iA_{i+1} and A_i'' on side A_iA_{i+2} be such that $A_i'A_i'' \perp OA_i$ and γ is the A_i–excircle of $\Delta A_iA_i'A_i''$ where $A_4 = A_1$, $A_5 = A_2$. Prove that

(a) $A_1'A_1''\cdot A_2'A_2''\cdot A_3'A_3'' = \dfrac{4a_1a_2a_3}{(a_1+a_2+a_3)^2}\cdot r^2$

(b) $A_1'A_1'' + A_2'A_2'' + A_3'A_3'' = \dfrac{a_1^2+a_2^2+a_3^2}{a_1a_2a_3}\cdot IK^2 + \dfrac{3a_1a_2a_3}{a_1^2+a_2^2+a_3^2}$